King's class

MA 120 Calculus + Its Applications
Final Exam Review Guide Parts 3+4
Solutions

Part 3

C ① $C(x) = 56x + 4000$ $x = \#$ of calculators

$C(1600) = 56(1600) + 4000$

$= \$93,600$

A ② Initial Value $= \$80,000$ $x = \#$ of years

$\$16,000$/year (depreciation)

i) $\boxed{f(x) = 80,000 - 16,000x}$

ii) $f(2) = 80000 - 16,000(2)$

$= 80,000 - 32,000$

$= \boxed{\$48,000}$

iii) $0 = 80,000 - 16,000x$

$+16,000x \qquad +16,000x$

$16,000x = 80,000$

$\frac{16,000x}{16,000} = \frac{80,000}{16,000}$

$\boxed{x = 5 \text{ yrs.}}$

A (3) $\dfrac{\text{Cost}}{\text{part}} = \dfrac{y}{x}$ $\quad x = [\overset{a}{100}, \overset{b}{400}]$

$(\underset{\substack{\text{parts}\\x}}{100}, \underset{\substack{\text{cost}\\y}}{\$300})$ $\quad (\underset{\substack{\text{parts}\\x}}{400}, \underset{\substack{\text{cost}\\y}}{2700})$

$$\text{average cost} = m = \frac{f(b) - f(a)}{b-a}$$

$$= \frac{2700 - 300}{400 - 100} = \frac{2400}{300} = 8$$

$\therefore$ $\boxed{\$8.00/\text{part}}$

A (4) Fixed Cost = \$24,000
Variable Cost = \$33/pair
price = \$101/pair

$C(x) = 24{,}000 + 33x$
$R(x) = 101x$
Break Even: $R(x) = C(x)$

$$101x = 24000 + 33x$$
$$-33x \qquad\qquad -33x$$
$$\frac{68x}{68} = \frac{24000}{68}$$
$$x \approx 352.9411$$

∴ So, the company needs to sell $\boxed{353 \text{ pairs}}$ to break even.

B (5) Fixed Cost = \$520
Variable Cost = \$38 / jacket

$$\boxed{C(x) = 520 + 38X}$$

Part 4

B (6) $C'(x) = 0.0006X^2 - 0.4X + 80$
total Cost = integrate

$$\int_{301}^{400} (0.0006X^2 - 0.4X + 80)\,dx$$

$$= \frac{0.0006X^{2+1}}{2+1} - \frac{0.4X^{1+1}}{1+1} + \frac{80X^1}{1}\Bigg]_{301}^{400}$$

$$= \frac{0.0006X^3}{3} - \frac{0.4X^2}{2} + 80X\Bigg]_{301}^{400}$$

$$= 0.0002X^3 - 0.2X^2 + 80X\Big]_{301}^{400}$$

$$= \left[0.0002(400)^3 - 0.2(400)^2 + 80(400)\right] -$$

$$\left[0.0002(301)^3 - 0.2(301)^2 + 80(301)\right]$$

$$= [12{,}800 - 32{,}000 + 32{,}000] -$$

$$[5454.1802 - 18120.2 + 24080]$$

$$= [12800] - [11413.9802$$

$$= 1386.0198 \approx \boxed{\$1386.02}$$

C ⑦ $v = t^2 + 6t + 7$

total distance = integrate

$$\int_0^4 (t^2 + 6t + 7)\,dt = \frac{t^3}{3} + \frac{6t^2}{2} + 7t \Big]_0^4$$

$$= \frac{t^3}{3} + 3t^2 + 7t \Big]_0^4$$

$$= \left[\frac{(4)^3}{3} + 3(4)^2 + 7(4)\right] - \left[\frac{0^3}{3} + 3(0)^2 + 7(0)\right]$$

$$= \left[\frac{64}{3} + 48 + 28\right] - [0 + 0 + 0]$$

$$= 97\tfrac{1}{3} \approx \boxed{97.3}\ \text{m}$$

C (8) $M'(t) = -0.006t^2 + 0.4t$

total words memorized = integrate

$$\int_0^{30} (-0.006t^2 + 0.4t)\,dt$$

$$= \frac{-0.006t^3}{3} + \frac{0.4t^2}{2}\Big]_0^{30}$$

$$= -0.002t^3 + 0.2t^2\Big]_0^{30}$$

$$= \left[-0.002(30)^3 + 0.2(30)^2\right] - \left[-0.002(0)^3 + 0.2(0)^2\right]$$

$$= [-54 + 180] - [0 + 0]$$

$$= \boxed{126 \text{ words}}$$

A (9) $P(t) = 4t^3 - 2t + 1 \quad [\overset{a}{0}, \overset{b}{5}]$

average Profit $= \frac{1}{b-a} \int_a^b P(t)\,dt$

$$\frac{1}{5-0} \int_0^5 (4t^3 - 2t + 1)\,dt$$

$$\frac{1}{5}\left[\frac{4t^4}{4} - \frac{2t^2}{2} + \frac{1t}{1}\right]_0^5$$

$\frac{1}{5}\left[t^4 - t^2 + t\right]_0^5$

$= \frac{1}{5}\left\{\left[(5)^4 - (5)^2 + (5)\right] - \left[(0)^4 - (0)^2 + (0)\right]\right\}$

$= \frac{1}{5}\left\{\left[625 - 25 + 5\right] - \left[0\right]\right\}$

$\frac{1}{5}\left[605\right] = \boxed{\$121/\text{day}}$

B ⑩ $C(t) = \frac{1}{2}(2t+1)^{-1/2}$ (Chain Rule)

rate of change = derivative

$C'(t) = \left(-\frac{1}{2}\right)\cdot\frac{1}{2}(2t+1)^{-1/2-1}(2)$

$= -\frac{2}{4}(2t+1)^{-3/2} = \dfrac{-1}{2(2t+1)^{3/2}}$

$C'(24) = \dfrac{-1}{2\left[2(24)+1\right]^{3/2}}$

$= \dfrac{-1}{2\left[48+1\right]^{3/2}} = \dfrac{-1}{2(49)^{3/2}} = \dfrac{-1}{2(7)^3}$

$= \dfrac{-1}{2(343)} = \boxed{\dfrac{-1}{686}}$

A ⑪ $f(x) = (2x^3+9)(4x^7-7)$ Prod. Rule: $f\cdot s' + s\cdot f'$

$f'(x) = (2x^3+9)(28x^6) + (4x^7-7)(6x^2)$

$= 56x^9 + 252x^6 + 24x^9 - 42x^2$

$= \boxed{80x^9 + 252x^6 - 42x^2}$

p.