King's class

MA 120 Calculus + Its Applications
Final Exam Review Part 2 of 4
Solutions

C ① $TS(x) = -2x^3 + 90x^2 + 1300x + 5000$

Point of Diminishing Returns = Point of Inflection = 2nd derivative

$TS'(x) = -6x^2 + 180x + 1300$

$TS''(x) = -12x + 180$

$0 = -12x + 180$

$\frac{12x}{12} = \frac{180}{12}$ $\quad x = 15$

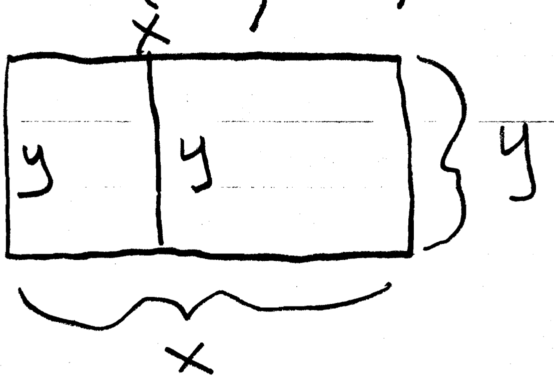
$TS(15) = -2(15)^3 + 90(15)^2 + 1300(15) + 5000$

$= -6750 + 20250 + 19500 + 5000$

$= 38{,}000$

Pt. of Diminishing Returns:
(15, 38,000)

A ②



Total Fence = 260 ft.

$2x + 3y = 260$

$A = x \cdot y$

Solve for y:

$$2x + 3y = 260$$
$$-2x \qquad -2x$$

$$\frac{3y}{3} = \frac{260}{3} - \frac{2x}{3}$$

$$y = \frac{260 - 2x}{3}$$

Substitute 'y' value into Area formula then take derivative to maximize area:

$$A = x\left(\frac{260-2x}{3}\right) = \frac{260x - 2x^2}{3}$$

$$A = \frac{260x}{3} - \frac{2x^2}{3}$$

$$A' = \frac{260}{3} - \frac{4x}{3}$$

$$0 = \frac{260}{3} - \frac{4x}{3}$$

$$\frac{4x}{3} = \frac{260}{3} \text{ (cross mult.)}$$

$$3(4x) = 3(260)$$

$$12x = 780$$

$$\boxed{x = 65 \text{ ft}}$$

$$y = \frac{260 - 2(65)}{3} = \frac{130}{3} = \boxed{43\tfrac{1}{3}\text{ ft}}$$

$$\text{Max } A = (65)(43\tfrac{1}{3}) = 65\left(\frac{130}{3}\right)$$

$$\boxed{\text{Max } A = 2816\tfrac{2}{3}\text{ ft}^2}$$

C ③ Relative Maximum (1st derivative)

$$h = -0.002d^2 + 0.6d + 6.0$$
$$h' = -0.004d + 0.6$$
$$0 = -0.004d + 0.6$$
$$0.004d = 0.6$$
$$d = 150 \text{ ft. horizontally}$$

$$h = -0.002(150)^2 + 0.6(150) + 6.0$$
$$= -45 + 90 + 6$$
$$= 51 \text{ ft. vertically}$$

$$\boxed{\text{Rel. Max } (150, 51)}$$

A ④ $R(x) = 500x - 0.01x^2$
$C(x) = 120x + 100{,}000$

Profit = Revenue − Cost
$P(x) = (500x - 0.01x^2) - (120x + 100{,}000)$
$= 500x - 0.01x^2 - 120x - 100{,}000$
$= -0.01x^2 + 380x - 100{,}000$

Max Profit = $P'(x)$
$P'(x) = -.02x + 380$
$0 = -.02x + 380$
$\frac{.02x}{.02} = \frac{380}{.02}$
$x = 19{,}000$ guages

$P(19{,}000) = -0.01(19000)^2 + 380(19000) - 100000$
$= -3610000 + 7220000 - 100000$
$= \boxed{\$3{,}510{,}000}$

Max Profit of \$3,510,000 when 19,000 guages are sold.

⑤ $C = 15s^2 - 6s + 1800$

Min. Cost = $C'(s)$

$$C'(s) = 30s - 6$$

$$0 = 30s - 6$$

$$\frac{6}{30} = \frac{30s}{30}$$

$$\frac{1}{5} = s$$

$$\boxed{s = 0.2} \text{ mHz}$$

The cost is a **minimum** when the processor speed is 0.2 mHz